## Localization Length in Anderson Insulator with Kondo Impurities

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The localization length,  $\xi$ , in a 2-dimensional Anderson insulator depends on the electron spin scattering rate by magnetic impurities,  $\tau_s^{-1}$ . For antiferromagnetic sign of the exchange, the time  $\tau_s$  is itself a function of  $\xi$ , due to the Kondo correlations. We demonstrate that the unitary regime of localization is impossible when the concentration of magnetic impurities,  $n_{\rm M}$ , is smaller than a critical value,  $n_c$ . For  $n_{\rm M} > n_c$ , the dependence of  $\xi$  on the dimensionless conductance, g, is reentrant, crossing over to unitary, and back to orthogonal behavior upon increasing g. Sensitivity of Kondo correlations to a weak parallel magnetic field results in a giant parallel magnetoresistance.

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Introduction. Scaling theory of localization [1] renders predictions for the dependence of the localization length,  $\xi$ , on the dimensionless conductance  $g = k_{\rm F} l$ , where  $k_{\rm F}$  is the Fermi momentum, and l is the mean free path. In two dimensions, in the absence of a magnetic field,  $\xi(g)$  increases with g as  $\exp(\frac{\pi g}{2})$ . Spin-flip processes caused by magnetic impurities affect the  $\xi(g)$  growth. The underlying reason is the suppression of the destructive interference, which, without magnetic impurities, facilitates localization. The quantitative measure of this suppression is the spin-flip scattering time,  $\tau_s$ , defined as

$$\frac{1}{\tau_s} = n_{\rm M} \sigma_{sf} v_{\rm F},\tag{1}$$

where  $n_{\rm M}$  is the concentration of magnetic impurities,  $\sigma_{sf}$  is the scattering cross section for a single magnetic impurity, and  $v_{\rm F}$  is the Fermi velocity. The growth of  $\xi(g)$  is significantly accelerated by magnetic impurities when the conductance is high enough, so that  $\xi(g)$  exceeds the spin–flip length,  $(D\tau_s)^{1/2}$  [2,3], where  $D=v_{\rm F}^2\tau/2$  is the diffusion coefficient, and  $\tau$  is the scattering time. For higher g-values  $\xi(g)$  rapidly crosses over to  $\exp(\pi^2g^2)$ -behavior, corresponding to the unitary ensemble. The suppression of the weak localization correction due to magnetic impurities was first pointed out in Ref. [4]. The fact that the first non-vanishing term in the scaling function is  $\sim 1/g^2$ , suggesting the orthogonal–unitary crossover, was established in Refs. [5,6].

The exact form of the dependence  $\ln \xi(g)$  in the presence of the spin–flip scattering cannot be inferred from the scaling theory. Still it is possible to utilize the approach of Lerner and Imry [7], who considered the orthogonal–unitary crossover with magnetic field, and apply it to the case of the spin–flip scattering. This yields

$$\ln\left(\frac{\xi}{l}\right) = \frac{1}{2}\ln\left(\frac{\tau_s}{\tau}\right) + \left[\pi g - \ln\left(\frac{\tau_s}{\tau}\right)\right]^2,\tag{2}$$

Eq. (2) emerges upon integration of the scaling equation from the largest length,  $\xi$ , to the smallest length, l. In course of integration, the unitary form of the  $\beta$ -function must be used within the interval  $\left[\xi, (D\tau_s)^{1/2}\right]$  and the orthogonal form within the interval  $\left[(D\tau_s)^{1/2}, l\right]$ . A crossover equation similar to Eq. (2) emerges from the calculation of the  $\beta$ -function following Ref. [8], but with  $\tau_s$  kept finite.

In previous considerations it was implicit that magnetic impurities are free to flip their spins. More precisely, the ability to flip spins was assumed independent of the degree of localization of the surrounding electrons. On the other hand, for antiferromagnetic sign of the exchange, it is evident that at zero temperature the spin-flip scattering is completely suppressed due to the presence of the Fermi sea. Indeed, as the temperature is lowered, the screening of spin of the magnetic impurity by the Fermi sea electrons drives the impurity into the Kondo state, in which only spin-conserving scattering is possible [9,10]. Thus, at zero temperature, the spin-flip scattering rate is expected to vanish, and hence, no orthogonal–unitary crossover in  $\xi(g)$ .

Our main point here is that the above scenario does not apply to the Anderson insulator. The reason for this is the following. In the localized regime, a single magnetic impurity "communicates" only with surrounding electrons within a spatial domain  $\sim \xi$ . In other words, the magnetic impurity in the insulating environment can be viewed as a Kondo-box [11] of size  $\sim \xi$ . As a result, the ground state of this impurity is determined by the ratio of two energies, namely  $T_{\rm K}$  – the Kondo temperature in the bulk, and  $\Delta_c$  – the level spacing in the box. In the case of the Anderson insulator, we have  $\Delta_c = (\nu \xi^2)^{-1}$ , where  $\nu$  is the density of states [12]. This leads us to the conclusion that the spin-flip scattering rate at zero temperature can be obtained from the finite—temperature expression [9,10] with temperature, T, replaced by  $\Delta_c$ .

Therefore, the time,  $\tau_s$ , which determines the localization length,  $\xi$ , through Eq. (2), is *itself* a function of  $\xi$ . With  $\tau_s$  being  $\xi$ -dependent, Eq. (2) becomes an *equation* for the localization length. As we will demonstrate below, this leads to a nontrivial behavior of  $\xi(g)$ .

Localization length. In order to find  $\xi(g)$  from Eq. (2), the dependence  $\tau_s(\xi)$  should be specified. This can be done in two limiting cases

(i) underdeveloped Kondo regime,  $T_{\rm K} < \Delta_c$ . In this limit the leading logarithm approximation [13,14] applies. Then, for the spin–flip scattering cross section, we can use the expression derived with leading logarithm accuracy [15,16] and replace  $T \to \Delta_c$ . For the impurity spin S = 1/2 this yields  $\sigma_{sf}^{-1} = 2k_{\rm F} \left[1 + \frac{4}{3\pi^2} \ln^2 \left(\Delta_c/T_{\rm K}\right)\right]$ . It is convenient to rewrite the above expression in terms of the ratios  $\tau_s/\tau$  and  $\xi/l$  that enter the crossover equation Eq. (2). Using Eq. (1), we obtain

$$\frac{\tau_s}{\tau}\Big|_{\Delta_c \gg T_{\rm K}} = \frac{4\pi n_e}{gn_{\rm M}} \left\{ 1 + \frac{4}{3\pi^2} \ln^2 \left[ \left(\frac{\xi}{l}\right)^{-2} \left(\frac{2\pi E_{\rm F}}{g^2 T_{\rm K}}\right) \right] \right\}, \quad (3)$$

where  $n_e = k_{\rm F}^2/2\pi$  is the concentration of electrons, and  $E_{\rm F} = \hbar k_{\rm F} v_{\rm F}/2$  is the Fermi energy.

(ii) fully developed Kondo regime,  $T_{\rm K} > \Delta_c$ . In this regime the spin of the magnetic impurity is screened, so that the spin-flip scattering is suppressed. The dependence of  $\tau_s$  on  $\xi$  can be inferred from the following reasoning. In a homogeneous system at temperature  $T \ll T_{\rm K}$  the full scattering cross section for an electron with energy  $\sim T$  by a magnetic impurity differs from the unitary value by a fraction  $\sim (T/T_{\rm K})^2$ . This deviation comes from two, comparable to each other, spin-conserving and spin-flip contributions. In our case the temperature is zero, but the magnetic impurity is effectively located in the "box" of a size  $\sim \xi$  with discrete electron levels spaced in energy by  $\Delta_c \ll T_{\rm K}$ . Using  $\Delta_c$  instead of T as the smallest energy scale, yields the following expression for  $\tau_s$  in the fully developed Kondo regime

$$\frac{\tau_s}{\tau}\Big|_{\Delta_c \ll T_{\rm K}} = \frac{4\pi}{g} \left(\frac{n_e}{n_{\rm M}}\right) \left(\frac{T_{\rm K}}{\Delta_c}\right)^2 = \frac{g^3 n_e}{\pi n_{\rm M}} \left(\frac{\xi^2 T_{\rm K}}{l^2 E_{\rm F}}\right)^2, \tag{4}$$

where the coefficient  $4\pi$  in the first identity is chosen in such a way that Eqs. (3) and (4) match at  $\Delta_c \approx T_{\rm K}$ . Replacement of T by  $\Delta_c$  in Eq. (4) warrants a more detailed discussion. Indeed, after being placed into the box, the ability of the magnetic impurity to flip the spin depends on the actual disorder realization within the box via the parity effect [11]. In other words, the deviation from complete screening fluctuates randomly between zero and  $\Delta_c/T_{\rm K}$ . Finite T, on the other hand, causes the deviation from the complete screening by the amount  $\sim T/T_{\rm K}$ . The similarity between the situations with finite T and finite  $\Delta_c$  is furthered by the observation that formation of the Kondo state in the box with even number of electrons involves partial occupation of the

first level above the Fermi level [11]. The same happens when the temperature is of the order of  $\Delta_c$ .

The dependence  $\tau_s(\xi)$  defined by Eqs. (3), (4) is depicted schematically in Fig. 1a. The minimum corresponds to  $\xi = \xi_{\rm K}$ , where the length  $\xi_{\rm K}$  is defined as

$$\xi_{\rm K} = \left(\frac{E_{\rm F}}{n_e T_{\rm K}}\right)^{1/2}.\tag{5}$$

At  $\xi = \xi_{\rm K}$  we have  $\tau_s/\tau \approx 4\pi n_e/gn_{\rm M}$ . Fig. 1a allows us to draw certain conclusions about the behavior of the localization length as a function of the conductance. Indeed, if the minimum lies above the bisector  $\tau_s/2\tau = (\xi/l)^2$ , then the  $\xi(g)$ - dependence remains "orthogonal",  $\ln(\xi/l) = \pi g/2$ , at any g. The corresponding condition can be rewritten as  $n_{\rm M} < n_c$ , where

$$n_c = \frac{n_e}{\pi} \left( \frac{T_K}{E_F} \right) \ln \left( \frac{E_F}{T_K} \right), \tag{6}$$

is the critical concentration of magnetic impurities. When  $n_{\rm M}$  exceeds  $n_c$ , the line  $\tau_s/2\tau = (\xi/l)^2$  intersects the  $\tau_s/\tau$  curve at two points,  $\xi_1/l = \exp{(\pi g_1/2)}$  and  $\xi_2/l = \exp{(\pi g_2/2)}$ . Therefore, the orthogonal behavior of  $\xi(g)$  holds within the domains  $g < g_1$  and  $g > g_2$ , as illustrated in Fig. 1b. With logarithmic accuracy the values  $g_1$  and  $g_2$  are given by

$$g_1 = \frac{1}{\pi} \ln \left( \frac{2\pi n_e}{n_{\rm M}} \right), \quad g_2 = \frac{1}{\pi} \ln \left[ \left( \frac{\pi n_{\rm M}}{n_e} \right) \left( \frac{E_{\rm F}}{T_{\rm K}} \right)^2 \right]. \quad (7)$$

In the domain  $g_1 < g < g_2$  the dependence  $\xi(g)$  takes different forms within the intervals  $g_1 < g < g_{\rm K}$  and  $g_{\rm K} < g < g_2$ , as shown in Fig. 1b. The crossover conductance,  $g_{\rm K}$ , is determined from Eq. (2) by substituting  $\xi = \xi_{\rm K}$  and using Eq. (3) for  $\tau_s/\tau$ . This yields

$$g_{\rm K} = \frac{1}{\pi} \left\{ \ln \left( \frac{4\pi n_e}{n_{\rm M}} \right) + \frac{1}{2^{1/2}} \ln^{1/2} \left( \frac{n_{\rm M} E_{\rm F}}{2n_e T_{\rm K}} \right) \right\}.$$
 (8)

In the interval  $g_1 < g < g_{\rm K}$  the  $\xi(g)$  dependence follows Eq. (2) (see Fig. 1b). Further growth of  $\xi$  with conductance is slow due to the suppression of the spin–flip scattering in the fully developed Kondo regime. The analytical form of  $\xi(g)$  in the interval  $g_{\rm K} < g < g_2$  can be found from Eq. (2) upon substituting Eq. (4) for  $\tau_s$  and solving for  $\xi$ . We obtain

$$\ln\left(\frac{\xi}{l}\right) = \frac{\pi g}{4} + \frac{1}{4}\ln\left(\frac{\pi n_{\rm M} E_{\rm F}^2}{n_e T_{\rm K}^2}\right) = \frac{\pi (g + g_2)}{4}.$$
 (9)

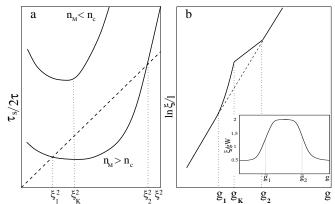


FIG. 1. (a) Dependence of the dimensionless spin scattering time on the localization length (in the units of l) as defined by Eqs. (3), (4) is depicted schematically for different concentrations of magnetic impurities,  $n_{\rm M}$ ; (b) localization length,  $\xi$ , is plotted versus dimensionless conductance for  $n_{\rm M} > n_c$ . For  $g < g_1$  and  $g > g_2$  the straight line reflects the "orthogonal" behavior  $\ln{(\xi/l)} = \pi g/2$ . In the interval  $g_1 < g < g_{\rm K}$  localization length follows the crossover formula Eq. (2). Novel regime Eq. (9) due to the Kondo effect takes place within the interval  $g_{\rm K} < g < g_2$ . The inset shows the dimensionless localization length in a quasi-one-dimensional geometry, plotted schematically versus dimensionless conductance for  $n_{\rm M} > \tilde{n}_c$ .

Eq. (9) is our central result. It describes the intermediate behavior of the localization length, when it is much smaller than the "unitary" value, but much larger than the "orthogonal" value (see Fig. 1b). The slope,  $\partial \ln \xi/\partial g = \pi/4$  in this regime is 2 times smaller than in the orthogonal regime. The factor 2 originates from the  $\xi$ -dependence of the r.h.s. of Eq. (4) and reflects the underlying physics, namely, that the ability of the Kondo impurity to flip the spin is governed by the level spacing in the Kondo–box [11].

We found that the spin-flip induced enhancement of the length  $\xi$  occurs when the concentration of magnetic impurities exceeds  $n_c$  given by Eq. (6). At  $n_{\rm M}=n_c$ , the condition  $\tau_s/2\tau=(\xi/l)^2$  is satisfied for  $\xi=\xi_{\rm K}$ . For consistency of our consideration the number of magnetic impurities within the area  $\xi_{\rm K}^2$  must be large. Indeed, from Eqs. (6), (5) we find  $n_c \xi_{\rm K}^2 \sim \ln{(E_{\rm F}/T_{\rm K})} \gg 1$ .

Eqs. (6), (5) we find  $n_c \xi_{\rm K}^2 \sim \ln{(E_{\rm F}/T_{\rm K})} \gg 1$ . Quasi-1D geometry. The above consideration can be easily extended to the quasi-one-dimensional geometry, i.e. a wire of a width W. The difference between the orthogonal and unitary regimes is less pronounced in this geometry. Namely, with increasing g the localization length crosses over from  $\xi(g) = gW/2$  to  $\xi(g) = 2gW$ . The crossover equation, analogous to Eq. (2), is known [17]. Analysis of this equation is very similar to the 2D case. In this analysis the expressions (3), (4) for  $\tau_s/\tau$  must be modified since for a wire the level spacing in the Kondo-box takes the form  $\Delta_c = (\nu \xi W)^{-1}$ . We will outline the results of this analysis. Presence of magnetic impurities affects  $\xi(g)$  if the impurity concentration,  $n_{\rm M}$ , exceeds  $\tilde{n}_c \sim n_e (T_{\rm K}/E_{\rm F})$ . For  $n_{\rm M} > \tilde{n}_c$  the  $\xi(g)$  dependence is depicted schematically in Fig. 1b, inset. The first crossover at  $\tilde{g}_1 = \left(n_{\rm M}W^2\right)^{-1}$  is a conventional orthogonal–unitary crossover. The second crossover back to the orthogonal result occurs at  $\tilde{g}_2 = n_{\rm M} \left(E_{\rm F}/n_eWT_{\rm K}\right)^2$ , when the Kondo regime is fully developed.

The effect of RKKY exchange. As we have established above, the localization length exceeds the "orthogonal" value only when  $n_{\rm M}>n_{\rm c}$ . The upper limit on  $n_{\rm M}$ , for which our consideration applies, comes from the indirect RKKY interaction, which leads to the strong correlation of spins of neighboring magnetic impurities (spin glass) and, thus, precludes the formation of the Kondo states for individual impurities. In order to incorporate the RKKY interaction,  $E_{in}(r)$ , into the theory, we note that, even if on average this interaction exceeds  $T_{\rm K}$ , a certain fraction, f, of the impurities still remains in the Kondo regime. This fraction is equal to

$$f = \exp\left(-\pi n_{\rm M} r_{\rm K}^2\right),\tag{10}$$

where  $r_{\rm K}$  is determined from the condition  $E_{in}(r_{\rm K}) = T_{\rm K}$ . Then  $fn_{\rm M}$  is the concentration of the *isolated* magnetic impurities, for which the indirect interaction with all the other impurities is smaller than  $T_{\rm K}$ . Thus, taking into account the RKKY exchange between the magnetic impurities amounts to the replacement of  $n_{\rm M}$  in Eqs. (1), (3), (4), and (7–9) by  $fn_{\rm M} = n_{\rm M} \exp(-\pi n_{\rm M} r_{\rm K}^2)$ . The interval of  $n_{\rm M}$ , where the above consideration applies, is determined by the condition  $fn_{\rm M}=n_c$ . The latter product has a maximum, as a function of  $n_{\rm M}$ , at  $n_{\rm M} = \left(\pi r_{\rm K}^2\right)^{-1}$ . Thus, the interval of validity exists if  $\pi r_{\rm K}^2 < n_c^{-1}$ . It is known [18] that the disorder does not affect dramatically the decay law of the indirect exchange. Therefore, the dependence  $E_{in}(r)$  can be approximated as  $E_{in}(r) \approx E_{in}(0)/n_e r^2$ , where  $E_{in}(0)$  is the interaction at distances  $\sim n_e^{-1/2}$ . Using the expression Eq. (6) for  $n_c$ , the condition of validity can be rewritten in the form  $E_{in}(0) < E_{\rm F}/\ln{(E_{\rm F}/T_{\rm K})}$ . On the other hand, since the RKKY exchange integral also determines the exponent in the Kondo temperature, we can express  $E_{in}(0)$  through  $T_{\rm K}$ . This yields the estimate  $E_{in}(0) \sim E_{\rm F}/\ln^2(E_{\rm F}/T_{\rm K})$ . Thus, parametrically, the interval of validity of our theory exists, although the "large" parameter,  $\ln{(E_{\scriptscriptstyle \rm F}/T_{\scriptscriptstyle \rm K})}$ , is certainly not reliable.

Implications. Conventionally, the prime manifestation of the Kondo effect is a drop of the Drude conductivity with decreasing temperature. This behavior reveals the temperature dependence of the full scattering cross section from a Kondo impurity. Compared to this prominent effect, other aspects of the Kondo physics received much less attention in the literature. These more delicate aspects were addressed in connection with the pair breaking in superconductors [15], weak localization corrections to the conductivity of metallic films and wires [19], and, most recently, in connection with electron-electron interactions mediated by Kondo impu-

rities [20–23]. In the case of the Anderson insulator, the Kondo effect should manifest itself through the inelastic transport. Indeed, the Mott law for the low-temperature resistance of the Anderson insulator can be written as  $\ln R \sim (\Delta_c/T)^{1/3} = \left[\nu T \xi^2(g)\right]^{-1/3}$ , and applies at temperatures  $T \ll \Delta_c$ . As we demonstrated above, Kondo impurities govern the dependence  $\xi(g)$ . This immediately suggests that the resistance of the Anderson insulator is exponentially sensitive to a weak parallel magnetic field, which otherwise has no effect on electronic states. Indeed, for large enough  $\xi$ , so that  $\Delta_c \lesssim T_{\rm K}$ , a Zeeman splitting,  $\Delta_{\rm z} \sim \Delta_c$ , suppresses the Kondo correlations and, thus, causes a growth of  $\xi$ . In general, inelastic transport is a natural arena to test the scaling theory [1]. For example, a predicted doubling of  $\xi$  in quasi-1D systems in a weak perpendicular field [24] has been revealed through the giant negative magnetoresistance in the localized regime [25]. Correspondingly, we predict a giant negative magnetoresistance for  $g > g_{\rm K}$  and a strong positive magnetoresistance for  $g < g_{\text{\tiny K}}$ . When discussing inelastic transport, it is important to remember that our key equation Eq. (4) implies that a singleparticle localization length has a meaning only for electron energy  $\lesssim \Delta_c$ . This, however, does not restrict the applicability of the Mott law, since the typical activation energy  $\sim (\Delta_c T^2)^{1/3}$  is much smaller than  $\Delta_c$ . Note, that the spin-glass state of magnetic impurities in the Anderson insulator should be also sensitive to a weak parallel magnetic field. Since in the experiment,  $n_{\rm M}$  is never known precisely, there is a question as to how to distinguish the Kondo-state of impurities (with spins completely screened) from the glassy state (in which the spins are frozen). In the experimental paper [26], the mesoscopic measurements of the asymmetric (with respect to the reversal of magnetic field) component of the resistance allowed the authors to identify the spin-glass state. This asymmetric component originates from the fact that the time-reversal symmetry in the spin glass is broken.

Conclusion. Scaling theory of localization is a scheme that accounts for all orders in the disorder strength. The insightful observation [2,5,6] that interference effects and the localization which they cause, survive in the presence of the spin-flip processes, suggests that magnetic impurities can be included into the scaling theory through the crossover parameter  $(\tau_s/\tau)(\xi/l)^{-2}$ . In the present paper the many-body physics is incorporated into this scheme in a nonperturbative fashion, i.e. with full account for the interactions. This is achieved at the expense of the assumption that the Coulomb interaction between the electrons is screened due to, say, a gate electrode. The only type of interactions retained is the on-site Hubbard repulsion [27], which is responsible for the magnetic properties of an impurity, and gives rise to the Kondo physics in the presence of the Fermi sea. Retaining only the Hubbard repulsion, allows one to trace the interplay of the interference and interaction effects on a nonperturbative level. Our study suggests that this interplay results in the enhancement of localization.

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